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13. ABSTRACT *(Maximum 200 words)* This program is devoted to understanding fundamental process in actual combustion chambers through coordination of theory, analysis and experiment Theoretical work has been carried out in the framework of an approach based on a form of Galerkin's method. General unsteady motions are synthesized of modes. Spatial averaging produces a representation of the unsteady behavior in a combustion chamber as the time evolution of a system of coupled nonlinear oscillators, one for each mode Consequently, immediate advantage can be taken of the methods available in contemporary research on nonlinear dynamical systems. The experimental work has involved tes with a Rijke tube with the Caltech dump combustor developed and used in work funded by AFOSR over the past 12 years. Those tests have demonstrated that due the presence of hysteresis in the stability of oscillations in the dump combustor, suppression of the oscillations is possible over a wide range of equivalence ratio by pulsed injection of secondary fuel in the recirculation zone. We have shown that the behavior is related to a subcritical bifurcation in the dynamics of the recirculation zone and unsteady combustion associated with vortex shedding.

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Abstract

This is the Final Technical Report covering work performed during the period 1 April 1995 - 31 March 1996 under AFOSR Grant, Contact No. F49620-95-1-0272, titled "Modeling and Active Control of Nonlinear Unsteady Motions in Combustion Chambers." Dr. Mitat Birkan served as Program Manager. The emphasis in this program is placed on understanding fundamental processes in actual combustion chambers through close coordination of theory, analysis and experiment. Theoretical work and analysis have been carried out in the framework of an approach initiated many years ago, based on a form of Galerkin's method. General unsteady motions are synthesized of modes of the chamber. Spatial averaging then produces a representation of the unsteady behavior in a combustion chamber as the time evolution of a system of coupled nonlinear oscillators, one for each mode. Consequently, immediate advantage can be taken of the methods available in contemporary research on nonlinear dynamical systems. Several problems related to nonlinear combustion instabilities and their control have been investigated during the past year. The experimental work has involved simple tests with a Rijke tube, but more importantly with the Caltech dump combustor developed and used in work funded by AFOSR over the past 12 years. Probably the most significant experimental result of the present program has been the demonstration that due the presence of hysteresis in the stability of oscillations in the dump combustor, suppression of the oscillations is possible over a wide range of equivalence ratio by pulsed injection of secondary fuel in the recirculation zone. Although understanding of this phenomenon is presently incomplete, we have established that is related to a subcritical bifurcation in the dynamics of the recirculation zone and subsequent unsteady combustion associated with vortex shedding. Both theoretical and experimental work will continue on this problem.

1. Research Objectives

This work has been part of a larger continuing program at Caltech concerned with fundamental problems arising in the application of active control of unsteady motions in combustion chambers. We believe that considerable progress is required to reach a level of basic understanding sufficient to explain observed behavior, with or without control, and to provide the foundation for more rational design and application of active control. Despite the number of apparent successes of control of combustion instabilities in laboratory devices, those successes are virtually all partial (oscillations are not entirely eliminated) and there are no complete explanations why the amplitudes have been changed by the use of control. Thus there is, for example, no sound basis for scaling to full-scale devices in which, among other differences, the power densities of combustion processes and the levels of intrinsic noise are significantly greater. These are among the features of full-scale combustors that have not previously been addressed in research programs.

Broadly the subjects covered by this program include

1) linear and nonlinear acoustics;
2) oscillations in the presence of noise;
3) modeling nonlinear combustion processes; and
4) active control of combustion systems

More specifically, the topics actually addressed are:

1) modeling the Rijke tube and the dump combustor to produce results in forms suitable for applying the principles of feedback control;
2) analysis of nonlinear combustion instabilities with stochastic (noise) sources and linear or nonlinear combustion responses;
3) control of an unstable combustor containing time lags; and
4) active control of a combustor by using pulsed injection of a secondary fuel supply.

The four items are more closely related than may appear to be the case at first acquaintance, although we are far from completing the task of integrating the results. That is of course a long range objective, part of the foundation of understanding how to scale the methods of active control from the laboratory to operational devices.

The general program of active control of combustion systems, and problems of the behavior of unsteady nonlinear motions in combustion chambers benefits from support with three other grants: ENEL (Italian National power company), *Research on Problems Relating to Pressure Oscillations in Gas Turbine Powered Stationary Powerplants;* Advanced Gas Turbine Systems Research, Department of Energy, *NOx and CO Emissions Models for Gas-Fired, Lean Premixed Combustion Turbines;* and a Multidisciplinary University Research Initiative funded by BMDO and managed by ONR, *Investigations of Novel Energetic Materials to Stabilize Rocket Motors.* Portions of these programs are distinct from the effort reported here, but without the coordinated support in those areas that overlap it would be impossible to carry out this work.

2. Research Accomplishments

2.1 Modeling of the Rijke Tube and the Dump Combustor

Although there exist many reports of data taken with Rijke tubes, and a few attempts to provide analytical representations or models, understanding the observed behavior is seriously incomplete. Almost all analyses have been linear and, as usual with problems of combustion instabilities, the frequencies and mode shapes can be predicted quite well to first approximation. However, predictions of linear stability are poor at best* and successful predictions of amplitudes are nonexistent. Because the unsteady motions in a combustor are 'self-excited' not requiring an external agent, the amplitudes are limited only by the action of nonlinear processes. Hence one cannot claim to understand the unsteady behavior of a combustor without investigating

*In fact, as we have established in this program, the Caltech dump combustor, and probably other similar combustors, do not possess the property of linear instability: the process of exciting oscillations is inherently nonlinear.

nonlinear processes and determining the main reasons for the existence of limiting amplitudes (often having approximately the character of periodic limit cycles).

Another significant, and often ignored feature of unstable combustors, is the presence of several (and sometimes many) modes. One conceivable explanation is based entirely on the assumption that the system is linear. In that limit, there are two possible reasons that more than one mode appears:

1) one mode is unstable, but linearly coupled to other modes, causing them to be driven to finite amplitude; or

2) the driving mechanism, conversion of thermal energy to potential energy of fluid motions, is sufficiently strong over a broad range of frequencies to excite and sustain the various modes observed.

Neither explanation, of course, will provide any information about the amplitudes reached. The second proposal could be true if the driving mechanism really is broad-banded. There is no evidence that is true in any real combustor, although, admittedly, there is virtually no data except for solid rockets. In that case, the driving, expressed as a response or admittance function, normally has a peak that spans a frequency range covering only one or two modes. A Rijke tube driven by an electrical heater may present a different situation but the action of a heater has not been sufficiently well-established to prove that the unsteady transfer of energy to the flow is significant over the broad range of frequencies required to support the second explanation. It seems a reasonable first guess that most mechanisms for driving combustion instabilities by heat transfer or energy addition are not sufficiently strong to sustain oscillations over a broad range of frequencies.

The first explanation seems quite appealing, but can be dismissed if one accepts the results of the approximate analysis described in Section 2.2, an analysis that has been successful for many years in many applications to combustion instabilities. This matter has been examined in one publication supported by this grant, Culick (1996). The relevant result is the following.

Linear coupling (e.g. associated with processes in a flame) is proportional to some small parameter, say ε which itself is small because it is proportional to the Mach number of the average flow. The analysis referred to is based on a representation of the unstable motions as a system of coupled oscillators. It is a familiar result, easily demonstrated for the ease of two oscillators, that the effects of linear coupling appear in the order of ε^2. Both the frequency shift and, more importantly for the reasoning here, the growth constant for the driven oscillation are proportional to ε^2. Not only is ε^2 extremely small for most combustors, but the equations that have been used in support of this explanation, are valid only to first order in ε. Hence to give a complete and correct analysis of the effects of linear coupling, the equations must be re-derived to include contributions of order ε^2. The point is that the representation of combustion instabilities as a set of oscillators is more complicated than a simple analogy with the classical problem of coupled oscillators.

One of the basic assumptions in this work is that nonlinear processes are intrinsic to the behavior of motions in combustors. Therefore, the presence of several modes, even when only one is unstable, is a reflection of nonlinear coupling, due either to gasdynamics (known to be present) or to combustion. If the assumption is true, then there may be significant implications for successful multi-mode control of combustion instabilities but this matter remains open.

2.1.1 <u>Modeling Oscillations in the Rijke Tube</u>

The unstable modes in a Rijke tube (Figure 1) are purely longitudinal. Hence the problem of modeling is apparently easily solved. Plane waves, having appropriate speeds of sound in the two regions up and downstream of the heater, are matched at the heater. Boundary conditions at the entrance and exit require admittance functions representing the fact that both pressure and velocity fluctuations are nonzero at the ends. Those admittance functions can only be estimated, but the errors are probably not large and if the processes are assumed linear, these values have little effect on the qualitative features of nonlinear behavior. Of course, the values of the

admittance functions affect linear stability directly, and have a strong influence on the amplitudes of oscillations.*

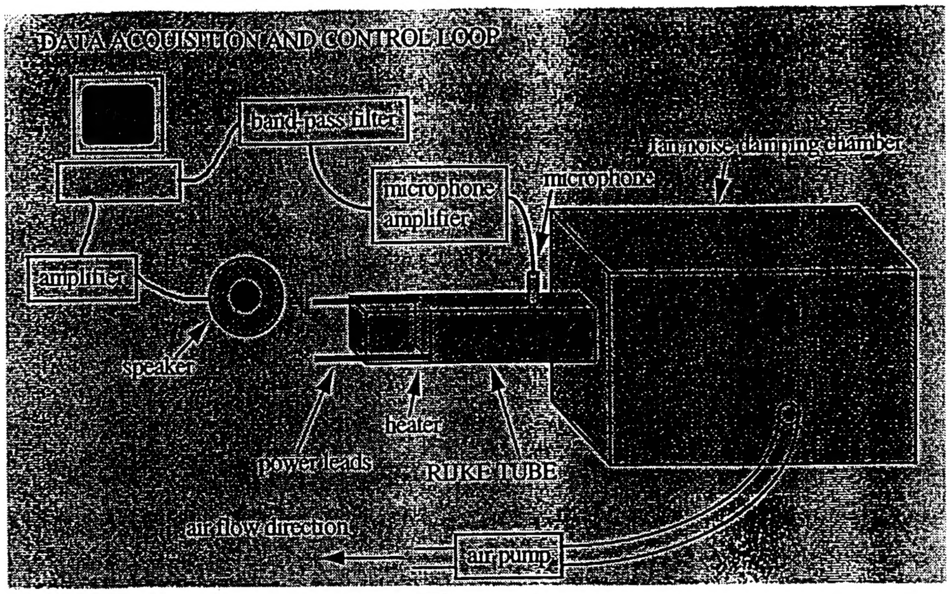

Figure 1

The truly difficult problem arises with determining the correct matching conditions at the heater or flame. In our experiments we have been using an electrical heater constructed as a grid. We have yet to settle on satisfactory matching conditions for the following reasons.

We assume that the tube is separated into two sections: cold section upstream of the heater (section 1), and hot section downstream of the heater (section 2). The mean quantities of the flow are constant in each section, but undergo an abrupt change through the interface. This becomes the familiar problem of heat addition in a one-dimensional duct. The solution is

$$\bar{p}_2/\bar{p}_1 = 1 - \gamma M_1^2(\lambda - 1) \equiv \beta, \tag{1}$$

$$\bar{u}_2/\bar{u}_1 = \bar{\rho}_1/\bar{\rho}_2 = \lambda, \tag{2}$$

$$\bar{T}_2/\bar{T}_1 = \lambda\beta. \tag{3}$$

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* An early result by Awad (1984) showed that if a combustor has uniform internal temperature and only two modes are considered, the time-averaged equations give $r_1/r_2=\sqrt{|\alpha_1/\alpha_2|}$ where r_i is the amplitude of the i^{th} mode. Similar results have been obtained for transverse modes. See Culick (1994) for a review.

The strength of the jump, $\lambda=\lambda(\bar{q},M_1)$, is a monotonically increasing function of $\bar{q}$, and $\lambda=1$ when $\bar{q}=0$.

The conservation equations and state equation for heat addition in a one-dimensional duct written to first order in small fluctuations are

$$\frac{\rho_2'}{\bar{\rho}_2}+\frac{u_2'}{\bar{u}_2}=\frac{\rho_1'}{\bar{\rho}_1}+\frac{u_1'}{\bar{u}_1}, \tag{4}$$

$$p_2'+2\bar{\rho}_2\bar{u}_2u_2'+\bar{u}_2^2\rho_2'=p_1'+2\bar{\rho}_1\bar{u}_1u_1'+\bar{u}_1^2\rho_1', \tag{5}$$

$$c_pT_2'+\bar{u}_2u_2'=c_pT_1'+\bar{u}_1u_1', \tag{6}$$

$$\frac{p_2'}{\bar{p}_2}-\frac{\rho_2'}{\bar{\rho}_2}-\frac{T_2'}{\bar{T}_2}=\frac{p_1'}{\bar{p}_1}-\frac{\rho_1'}{\bar{\rho}_1}-\frac{T_1'}{\bar{T}_1}. \tag{7}$$

Solving these simultaneously, and make use of the solution for the mean quantities and assuming that both $\bar{M}$ and M' are small leads to

$$p_2'=p_1'+O(M^2), \tag{8}$$

$$u_2'=\lambda u_1'+O(M^2), \tag{9}$$

$$T_2'=T_1'-(\gamma-1)(\lambda^2-1)\sqrt{\frac{\bar{T}_1}{\gamma R}}u_1'+O(M^3), \tag{10}$$

$$\rho_2'=\frac{1}{\lambda}\rho_1'+\frac{\lambda-1}{\lambda^2}\bar{\rho}_1\frac{T_1'}{\bar{T}_1}+O(M^2). \tag{11}$$

The fluctuating heat release is not included in this calculation; it will appear as a source term in the wave equation.

A problem arises when we also examine the differential equations governing the model Rijke tube.

$$\frac{\partial\rho}{\partial t}+\frac{\partial\rho u}{\partial x}=0, \tag{12}$$

$$\rho\frac{\partial u}{\partial t}+\rho u\frac{\partial u}{\partial x}=-\frac{\partial p}{\partial x}, \tag{13}$$

$$\frac{\partial p}{\partial t}+\gamma p\frac{\partial u}{\partial x}+u\frac{\partial p}{\partial x}=(\gamma-1)q. \tag{14}$$

The last equation is the energy equation written in terms of pressure. These equations together with the equation of state, $p=\rho RT$, adequately describe the system. We can derive the matching condition by integrating the

conservation equations across the region of discontinuity at the heater. We take the space of integration, Δx, to be infinitesimally small. Since all dependent variable are smooth functions of time, i.e., time derivatives are finite, as $\Delta x \to 0$, the integrals with time derivatives also go to zero. Integrating to first order in fluctuations and neglecting terms of second order in fluctuation and Mach number, we have

$$[\bar{\rho}u'] = 0 + O(M^2), \tag{15}$$
$$[p'] = 0 + O(M^2), \tag{16}$$
$$[u'] = 0 + O(M^2), \tag{17}$$

where $[\cdot]$ indicates jump across the heater. The first and third conditions obviously contradict each other. *It remains unresolved at this point as to why the differential equations give different results.* It should be noted that equations (9) and (15) are equivalent, and we believe this to be the correct matching condition for the fluctuating velocity. Work on this technical difficulty is continuing.

2.1.2 Modeling Oscillations in the Dump Combustor

Modeling unsteady motions in the dump combustor is a more elaborate form of the approach taken to analyze the Rijke tube. Neglecting combustion and the mean flow, Smith (1985), Sterling (1987), Zsak (1993) and Kendrick (1995) have all addressed this problem. If the temperature distribution is approximated reasonably well (even with no combustion) so the speeds of sound are close to actual values, then the linear results give frequencies and mode shapes approximating quite well the actual values.

In the present program, the JPC dump combustor sketched in Figure 2, has been modeled in fashion similar to the Rijke tube. An additional feature in the dump combustor is that it is composed of several different sections with differing cross-sectional areas. Since the observed oscillation is planar, a quasi-one-dimensional form of the approximate analysis, together with the matching conditions at an abrupt change in the mean quantities at the dump plane, can be used to simulate the system.

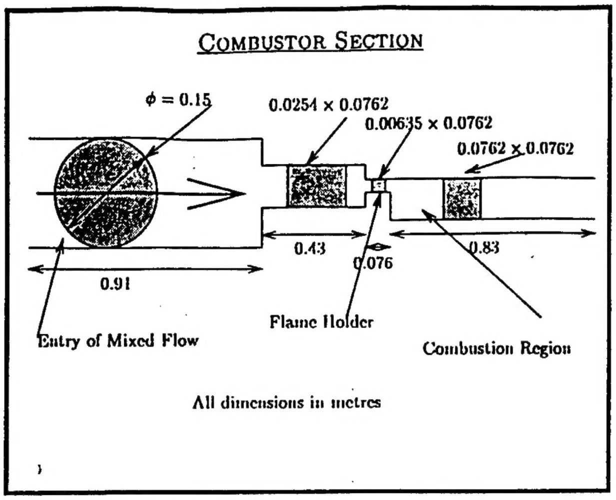


Figure 2

Using these dimensions and $\lambda=2.24$ corresponding to $\bar{T}_1=335\,K$ and $\bar{T}_2=750\,K$, we are able to calculate the natural oscillation modes of the chamber shown in Figure 2

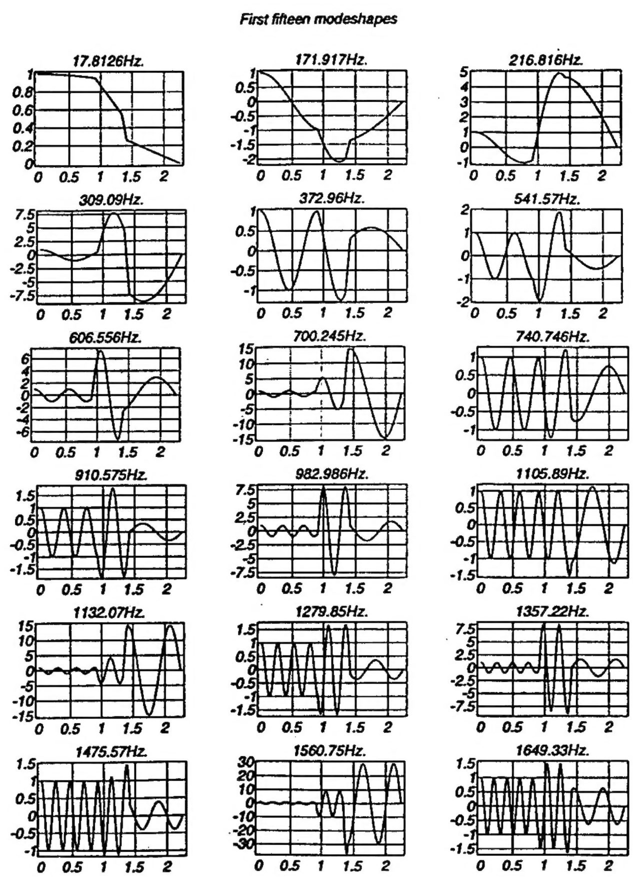


Figure 3

One surprising result is the existence of the bulk mode of oscillation at 17 hz. This mode has not been observed or reported in previous investigation. Since this frequency is so low, it may be obscured by background noise, or filtered by the data acquisition system. In simulation, the bulk mode introduces DC shift in higher modes. Also, when the bulk mode is included, the system does not seem to reach a limit cycle. The settling time becomes very long so that none of our simulations have reached a truly stable limit cycle (the limit cycle amplitude will shift given enough simulation time). We are currently using LSODE (Livermore Solver), which may have some difficulties with the jumps and nonlinearities in our simulation code. We are exploring other ODE solvers, e.g., Numerical Recipes.

We have also found a subcritical bifurcation in numerical simulation with nonlinear heat release. This behavior in the dump combustor has been observed by Knoop (1996). Work on comparison between the analytical and experimental results is in progress.

The most important aspect of the numerical work is in identifying the proper model describing the nonlinear heat release which affects the dynamics of the system. Current models, though crude, capture some of the essence of the problem. We are intending to enhance our numerical models to give quantitative agreement.

2.3 Analysis of Nonlinear Combustion Instabilities with Nonlinear Combustion Responses

For many years a puzzling unsolved problem has been the behavior of combustors subject to pulsed disturbances. If a combustor is linearly stable, then small disturbances, including sufficiently small pulses generated by external means, decay. However, it has long been known that in both liquid and solid rockets, sufficiently large pulses will often excite oscillations that develop into limit cycles. The earliest examples seem to have arisen with pulsing of liquid or gas rockets in the 1960's during development of engines for the Apollo vehicle. In the present program we have concentrated on the problem in solid propellant rockets.

Formally the problem consists in determining the conditions under which subcritical bifurcations will occur (Figure 6). During the past few years we have established beyond doubt, but without a formal proof, that nonlinear gasdynamics alone does not contain subcritical bifurcations (Culick 1994). The physical reason seems to be that the energy in the initial pulse is simply redistributed by nonlinear mode coupling and dissipated by the stable modes. A source of energy is required to sustain a motion in the presence of the losses.

A few years ago, Baum and Levine (1982 and other works) successfully showed numerically that a simple nonlinear representation of velocity coupling, really a kinematically nonlinearity associated with the combustion response, does contain subcritical bifurcations. They were able to produce remarkably good agreement between their calculations and the behavior of pulses observed both in laboratory tests at room temperature and in hot firings. Numerical calculations are suggestive but are difficult to use for obtaining understanding of the phenomenon, particularly in respect to learning what really causes pulsed instabilities to occur.

In his thesis, Burnley (1996) has examined essentially two representations of the nonlinear combustion response: the velocity-coupled model used by Baum and Eeé, and in slightly modified form by Green (1990); and a pressure-coupled model in which the nonlinearity arises from the Arrhenius factor in the gas-phase reactions. It's an interesting result that the pressure coupled model does not produce pulsed instabilities unless the burning solid is intrinsically unstable. Apparently the difference between the two cases is connected, in an unknown way, with the structure of the equations containing <u>both</u> nonlinear gasdynamics <u>and</u> nonlinear combustion.

Stable limit cycles subsequent to pulsing seem not to exist if the only nonlinear processes are gasdynamics <u>or</u> combustion: it is necessary to have both nonlinear contributions. The reason, of course, is that the motions treated are those of the coupled systems (combustion and gasdynamics); the character of the global dynamics must depend on the dynamics of the combustion processes and of the gasdynamics, but the ways in which the two

parts of the whole system work together are not understood. The analysis proceeds, briefly, in the following way.

The formulation begins with the conservation equations for two-phase flow. Those equations can be combined to give an equivalent form for a single medium characterized by the mass-averaged properties of the actual mixture. All flow variables are written as sums of average and fluctuating values. Eventually a nonlinear wave equation can be derived for the pressure fluctuation, with the corresponding boundary

$$\nabla^2 p' - \frac{1}{a^2}\frac{\partial^2 p'}{\partial t^2} = h \tag{18}$$

$$\hat{n}\cdot\nabla p' = -f \tag{19}$$

The functions h and f are nonlinear functions of the pressure and velocity fluctuations, as well as other flow variables. It is a good approximation to write the pressure fluctuation as a synthesis of classical acoustic modes $\psi_n(\bar{r})$ having time-dependent amplitudes $\eta_n(t)$:

$$p' = \bar{p}\sum_{m=1}^{\infty}\eta_m(t)\,\psi_m(\bar{r}) \tag{20}$$

Substitution of this representation in the left hand side of (18) and spatial averaging with ψ_n as a weighting function gives the set of ordinary differential equations for the amplitudes,

$$\frac{d^2\eta_n}{dt^2} + \omega_n^2\,\eta_n = F_n \tag{21}$$

where $\omega_n = a\,k_n$ and

$$F_n = -\frac{a^2}{\bar{p}E_n^2}\left\{\int h\psi_n dV + \oiint f\psi_n ds\right\} \tag{22}$$

After the functions h and f are inserted in (20), some manipulations lead to the formula for F_n valid to second order in fluctuations:

$$-\frac{\bar{p}E_n^2}{a^2}F_n = \bar{\rho}\int(\bar{u}\cdot\nabla u' + u'\cdot\nabla\bar{u})\cdot\nabla\psi_n dV + \frac{1}{a^2}\frac{\partial}{\partial t}\int(\bar{\gamma}p'\nabla\cdot\bar{u} + \bar{u}\cdot\nabla p')\psi_n dV$$

linear gasdynamics

$$+\bar{\rho}\int\left[u'\cdot\nabla u' + \frac{\rho'}{\bar{\rho}}\frac{\partial u'}{\partial t}\right]\cdot\nabla\psi_n dV + \frac{1}{a^2}\frac{\partial}{\partial t}\int(\bar{\gamma}p'\nabla\cdot u' + u'\cdot\nabla p')\psi_n dV \tag{23}$$

nonlinear gasdynamics

$$+\oint\bar{\rho}\frac{\partial u'}{\partial t}\cdot\hat{n}\psi_n dS - \int\left[\frac{1}{a^2}\frac{\partial\mathcal{P}'}{\partial t}\psi_n + \mathcal{F}'\cdot\nabla\psi_n\right]dV$$

linear and nonlinear surface processes — other contributions

This result is very general, subject of course to restrictions arising in steps from the primitive conservation equations to the approximate forms in which h and f appear. Properly interpreted, the representation (20) is not as serious a constraint as might appear to be the case. The approach taken here provides only a theoretical framework: explicit results can be obtained only after the processes other than gasdynamics are modeled. With care, the framework can be used for analyzing unsteady motions in any combustor.

For examining the consequences of a nonlinear combustion response in solid propellant rockets, we use the surface integral involving $\partial\bar{u}/\partial t$, plus the nonlinear gasdynamics, plus all linear processes, not specified explicitly, but all linear effects are contained in the parameters α_n, θ_n one pair for each mode. The set of nonlinear oscillator equations can be written for second order gasdynamics,

$$\ddot{\eta}_n + \omega_n^2\eta_n = 2\alpha_n\dot{\eta}_n + 2\omega_n\theta_n\eta_n - \sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\left[A_{nij}\dot{\eta}_i\dot{\eta}_j + B_{nij}\eta_i\eta_j\right] + \oint\bar{p}\frac{\partial\bar{u}'}{dt}\cdot\hat{n}\,dS \tag{24}$$

In the derivation of this equation, the velocity fluctuation in h and f has been approximated by the series

$$\bar{u}'(\bar{r}_1 t) = \sum_{m=1}^{\infty}\frac{\dot{\eta}_m(t)}{\gamma k_n^2}\nabla\psi_m(\bar{r}) \tag{25}$$

The two series (20) and (21) satisfy the classical linear acoustics equations term-by-term. Use of (25) in the formula for F_n will produce results correct to order $\overline{M}M'$ and M'^2. The reasons, and extension to include terms of order $\overline{M}M'^2$, are explained in Culick (1996).

As shown by (23) and (24), the boundary condition at the surface is expressed in terms of the velocity fluctuation, whereas analysis of the surface combustion processes leads in a natural way to results for the mass flux, $\dot{m}$. The relation between $(\partial\vec{u}'/\partial t)\cdot\hat{n}$ and $\left(\partial\dot{\vec{m}}^1/\partial t\right)\cdot\hat{n}$ is

$$\bar{\rho}\frac{\partial\vec{u}'}{\partial t}\cdot\hat{n}=\frac{\partial\dot{\vec{m}}'}{\partial t}\cdot\hat{n}-\rho'\frac{\partial\vec{u}'}{\partial t}\cdot\hat{n}-\frac{\partial\rho'}{\partial t}\left(\bar{\vec{u}}+\vec{u}'\right)\cdot\hat{n} \tag{26}$$

Baum and Levine (1982) proposed, and used, the model for the mass flux

$$\dot{m}=\dot{m}_{pc}\left[1+R_{rc}F(\vec{u})\right] \tag{27}$$

where $\dot{m}_{pc}$ depends on pressure only and leads to pressure coupling. Here, fluctuations of $\dot{m}_{pc}$ contribute only to α_n and θ_n. For illustration here we examine velocity coupling dependent on rectification only, with a threshold velocity, and $\dot{m}'$ is

$$\frac{\dot{m}'}{m}=\frac{R_{vc}}{a}\left|\vec{u}'-\vec{u}_t\right| \tag{28}$$

For the case of purely longitudinal motions, both $\vec{u}^1$ and $\vec{u}_t$ have only axial components and $F(u')=\frac{1}{a}\left|u'-u_t\right|$ has the form shown in Figure 4.

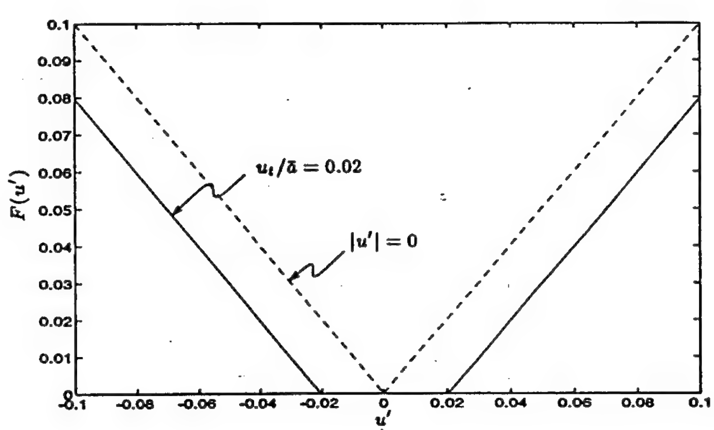


Figure 4

We have been investigating the consequences of nonlinear processes by applying a continuation method to construct bifurcation diagrams (Jahnke and Culick, 1994; Burnley 1996). Figure 5 shows one important conclusion from the present work: if gasdynamics is the only nonlinear process, then only supercritical bifurcations are found; the usual case for a linearly unstable system. If a combustion response of the form (10) provides the only nonlinear process, then no stable limit cycles are found.

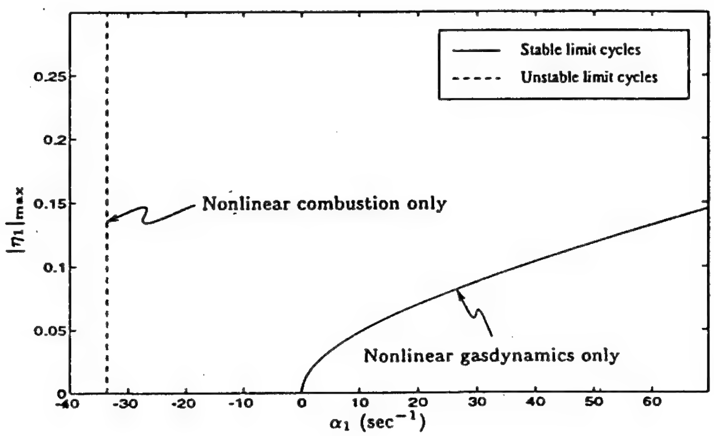


Figure 5

However, if both nonlinear combustion and nonlinear gasdynamics are included, then as Figure 6 shows, the form (27) does present the possibility for subcritical bifurcations providing the threshold velocity u_t is non-zero. Although we have not made sufficient calculations and comparison with the limited data available to show that the values of the parameters are realistic, these results already show that if there is a problem with pulsed instabilities in a motor, then probably the first place to look for a means of correction is in the response of burning to velocity fluctuations.

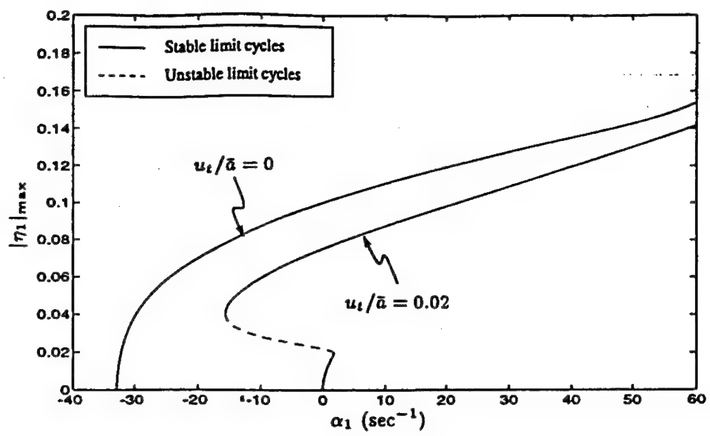


Figure 6

2.3 Analysis of Nonlinear Combustion Instabilities with Stochastic Sources

Unlike the case for the Rijke tube, the amplitude of pressure in a limit cycle reached in an actual combustor is never constant. Apart from changes on a relatively long time scale (many periods) due to changes of parameters, fluctuations normally occur, often over one or two periods. The true reasons for those apparently random changes of amplitude are not known – in fact the work in this program is the only attempt to address the matter. A logical speculation is that non-acoustic fluctuations in the flow, including 'combustion noise' and 'turbulence' generate pressure fluctuations that add to the organized oscillations associated with the acoustic waves. There are two problems that arise if this speculation is pursued.

1) identifying and modeling the dominant physical processes, to give representations of the stochastic sources; and
2) obtaining solutions to the governing equations containing the stochastic sources

So far we have not attempted to construct models of physical processes that might be responsible for noise generation. Rather, their random behavior appears to be that of white noise. The equations we use are constructed by beginning with the general result of applying Galerkin's method to a

combustor as outlined in Section 2.2. Random processes are taken into account by introducing the fundamental result (Chu and Kovasznay, 1958) than in the limit of vanishingly small amplitude; any disturbance in a compressible flow can be synthesized of three types of waves: acoustic waves which carry pressure but no entropy disturbances); vorticity waves, (free of pressure and entropy disturbances); and entropy or temperature waves (also possessing no pressure disturbances).

This approach, with some preliminary results, was reported by Culick et al (1991). Some technical errors of detail in the formulation have been corrected in the past year and calculations have been done for more complicated problems closer, we believe, to realistic cases (Burnley, 1996; Burnley and Culick, 1996).

The idea is to follow Chu and Kovasznay's discussion and write $\vec{u}'$ as the sum of acoustic, vortical and entropic contributions,

$$\vec{u}' = \vec{u}_a' + \vec{u}_\Omega' + \vec{u}_s' \tag{29}$$

where $\vec{u}_a'$ is given by (25) and the fluctuations $\vec{u}_\Omega$, $\vec{u}_s$ associated with vorticity and entropy waves are unspecified at this point. Our view is that to first approximation, the observed pressure field, irrespective of its origin is due entirely to acoustic waves, characterized by the variable p_a' and $\vec{u}_a'$. The acoustic field will in general consist of two parts: that associated with coherent motions, the oscillations identified in a combustion chamber as 'combustion instabilities'; and the field generated by stochastic sources included in the vorticity and entropy velocity fields $\vec{u}_r'$ and $\vec{u}_s'$ which required modeling not yet accomplished.

This approach bears a resemblance to Lighthill's theory of aerodynamic noise (Lighthill 1953) but is of course restricted to internal problems (Culick et al 1991). The great advantage of analyzing the internal problem is that natural modes and the expansions (20) and (25) are available. Nevertheless, the great problem eventually is to model the vorticity and entropy fields as sources of the stochastic or random pressure field.

Formula (23) shows explicit dependence of the 'force' F_n on the total velocity field; there may be additional contributions contained in p' and $\bar{\mathcal{F}}'$. The next step is substitution of (29) in (23) and collect terms. Because the contributions to F_n arising from gasdynamics have nonlinear terms, the results contain coupling between the three kinds of waves as well as terms involving squares of $\bar{u}'_a, \bar{u}'_\Omega$ and $\bar{u}'_s$ alone. Now the oscillator equations have the form

$$\ddot{\eta}_n + \omega_n^2 \eta_n = 2\alpha_n \dot{\eta}_n + 2\omega_n \theta_n \eta_n - \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} [A_{nij} \dot{\eta}_i \dot{\eta}_j + B_{nij} \eta_i \eta_j] + (F_n)^{\mathrm{NL}}_{\mathrm{other}} + \sum_{i=1}^{\infty} [\xi^v_{ni} \dot{\eta}_i + \xi_{ni} \eta_i] + \Xi_n, \quad (30)$$

where the $\xi^v_{n\zeta}$, ξ_{ni} and Ξ_n depend on the $\bar{u}'_r$ and $\bar{u}'_s$, presumably representing the stochastic sources referred to above.

The problem of modeling the stochastic has not yet been addressed in this work. Rather, the approach taken here has been to assume forms, guided by experimental observations. For simplicity, as a first step, we have taken the sources to be the limiting case of white noise having zero mean values. Figure 7 is an example of a simulation for a case in which the fundamental mode is unstable. The most satisfying feature is that the waveform and spectrum appear quite realistic, looking quite like actual test data. True quantitative comparison of predictions based on this theory with measurements is far off.

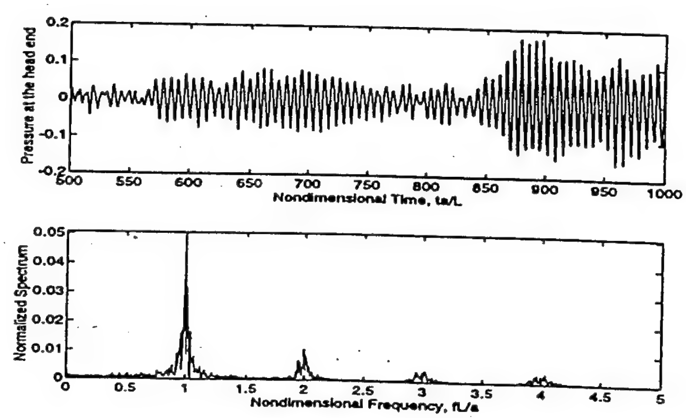


Figure 7

A different sort of problem is calculation of probability density functions for the amplitude of the fluctuations when the combustion response is nonlinear, having the form shown in Figure 4 which produces the

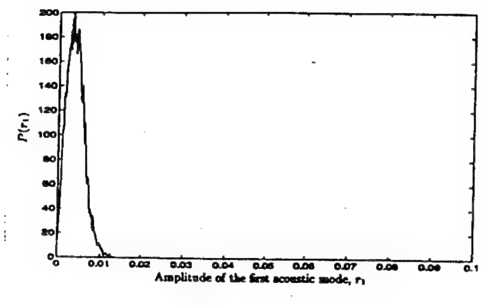
bifurcation diagram given in Figure 6. This problem has previously been examined by Clavin, Kim and Williams (1994) but in simplified form based on heuristic reasoning. The ;analysis in that work is very useful for gaining understanding of some central features of the problem. Because the authors assume that only a single mode is present, the formulation strictly cannot be applied to combustors and in fact it is not clear that the single nonlinear equation has any substantial connection to the physical situation. Nevertheless, that formulation contains two essential aspects: a subcritical bifurcation followed by a turning point (as here in Figure 6) and, of course, stochastic driving. Hence much of the general qualitative behavior is captured.

To gain an initial assessment of the validity of this approach we have carried out a few calculations for the following problem:

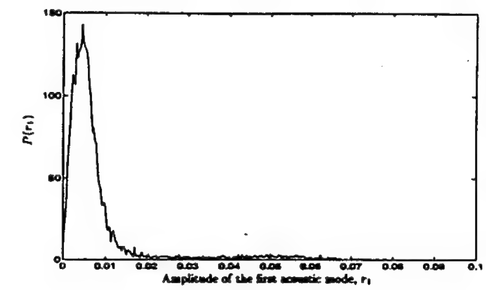
i) four modes included;
ii) stochastic sources in the lowest two modes
iii) nonlinear combustion response (Figure 54 giving the bifurcation diagram in Figure 6;
iv) growth constant α_1 varied from $-35s^{-1}$ to $25s^{-1}$;
v) the initial condition is a pulse of pressure, $p'/\bar{p}$ constant over the leftmost one-quarter of the chamber.

A Monte-Carlo method was used to solve the equations. Approximately 10,000 'experiments' were conducted for each value of a_1 and initial condition. For each experiment, after the flow has become developed (about 1000 periods of the fundamental mode) the amplitude of the acoustic modes are sampled. Histograms are then constructed from which the probability density function can be formed. Figures 8 shows results for a pulse having shape defined in v) above. (Burnley, 1996, Chapter 6). Perhaps the most obvious feature of this display is the appearance of a bimodal probability density function. The physical reason is the existence of two stable states in the range $-30s^{-1} < \alpha_1 < 0$. There is a peak in the probability density function in the vicinity of the stable quiescent state and near the stable limit cycle represented by the upper branch in Figure 6. For the first mode strongly unstable ($\alpha_1 = 10s^{-1}$, Figure 8) only a

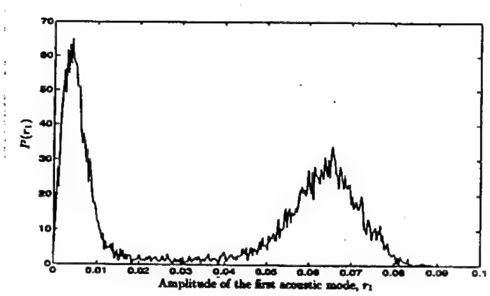
single peak is found because there is only one stable dynamical state, the periodic limit cycle. In the range $0<\alpha_1<5s^{-1}$, three peaks appear



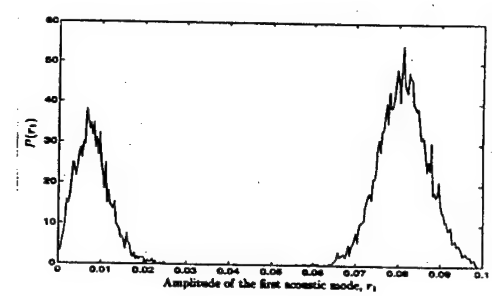
(a) $\alpha_1=-35s^{-1}$



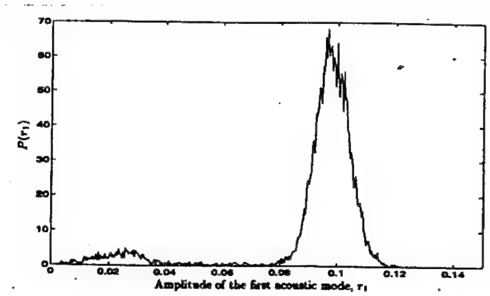
(b) $\alpha_1=-25s^{-1}$



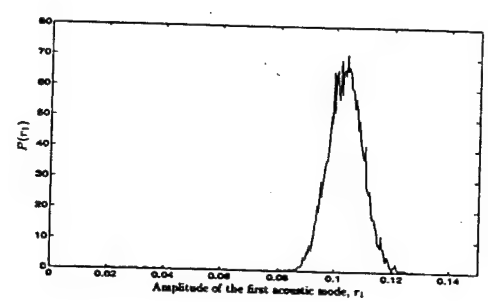
(c) $\alpha_1=-20s^{-1}$



(d) $\alpha_1=-10s^{-1}$



(e) $\alpha_1=5s^{-1}$



(f) $\alpha_1=10s^{-1}$

Figures 8

We have only just begun to explore the ramifications and practical implications of this approach. Further discussion has been given by Burnley (1996). One interesting point is the following. Suppose α_1 is in the range where a linearly stable system can be pulsed to give a stable limit cycle, here $-30<\alpha_1<5s^{-1}$. Consider the case $\alpha_1=-20s^{-1}$, say. If noise is not included, then a pulse having amplitude 0.02 is stable and the stable limit cycle cannot be excited. However, with noise included, if fluctuations add in the correct phase to the pulse, the total pressure perturbation can exceed the value given by the unstable branch in Figure 6 and the stable limit cycle is triggered. That is the reason that the second peak appears in the probability density function, in the vicinity of finite values of r_1 .

The chief motivation for investigating the influences of noise have been largely the need to understand the extent to which stochastic sources are significant in the development and nature of combustion instabilities. For example, a question raised occasionally is: can noise in a combustor excite and sustain coherent oscillations? The answer seems to be 'no,' that driving combustion processes are required. However, as indicated by the results just described for the behavior of pulses, noise can certainly have noticeable effects.

Another reason, not yet a pressing motivation, is related to future applications of active control to full-scale combustors. The possible problems of exercising control in the presence of high noise levels intrinsic to the system are not understood, and in fact have hardly been considered. However, it seems that eventually the matter must be addressed and to do so requires knowledge of the behavior of the combustor with stochastic sources. The work on the problems discussed in this section is part of the basis for modeling.

2.4 Control of an Unstable Combustor With Time Lag

Time delays are unavoidable in physical systems. Combustion chambers contain many sources of delays, of which the larger are probably due to transport of reactants and ignition delays. Chemical kinetics alone are

normally fast compared with other processes but there are circumstances when the characteristic times of fluid dynamics are comparable with those associated with the chemical dynamics. In the context of control, sensor and actuator delays cannot be ignored.

It is a fundamental principle in the subject of controls that time delays inevitably tend to destabilize a system. Hence in efforts to develop the use of active feedback control to combustion systems it is essential that the influences of time delays be examined and understood. For example, Kendrick (1995) has reported ignition delays of the order of the period of the fundamental mode of the Caltech dump combustor, a large delay if one is interested in controlling that mode.

Control of a system having a time delay is rendered even more difficult if the system alone is unstable. A simple example, the simplest possible, makes the point. Consider a single oscillator having undamped natural frequency Ω. Although we know that a single-mode approximation is not a faithful representation of a combustion instability, we ignore the inaccuracy and assume that a combustion instability can be modeled as a single mode, unstable with growth constant α due to the intrinsic processes. Suppose in addition that control is to be exerted by a secondary fuel supply producing the rate of energy addition $\dot{q}$ which then appears as the source $\partial\dot{q}/\partial t=\ddot{q}$ in the oscillator equation:

$$\ddot{\eta}-2\alpha\dot{\eta}+\Omega^2\eta=\ddot{q} \tag{31}$$

We assume that the rate energy addition is controlled by sensing the pressure and using it to generate a control signal but with a time delay τ

$$\dot{q}(t)=k\{\eta(t-\tau)\} \tag{32}$$

The Laplace transform of equations (14) and (15) gives

$$\frac{N(s)}{Q(s)}=\frac{s}{s^2-2\alpha s+\Omega^2}=H(s) \tag{33}$$

$$Q(s)=-K(s)e^{-\tau s}N(s) \tag{34}$$

where K(s) is the transform of $k\{n(t)\}$ and $N(s)$ is the transform of $\eta(t)$. Figure 9 is a block diagram of the system.

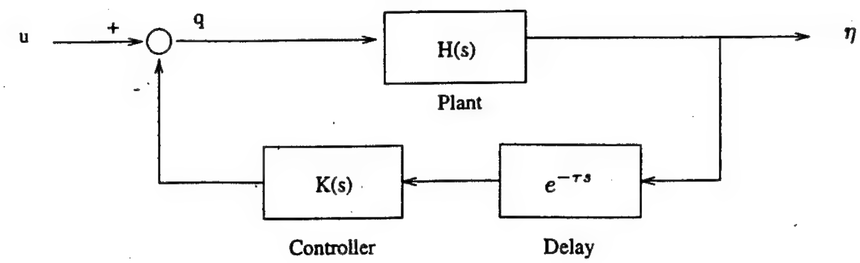


Figure 9

The closed-loop transfer function is

$$\frac{N(s)}{U(s)}=\frac{H(s)}{1+K(s)H(s)e^{\tau s}} \tag{35}$$

Because the system itself has negative damping, it is open-loop unstable: H(s) has two unstable poles in the right half plane. If the time delay is zero, the system can be stabilized, for example, by setting K(s) equal to a suitable constant value. The denominator of (35) is

$$1+K(s)H(s)\pm1+K(s)\frac{s}{s^2-2\alpha s+\Omega^2}=\frac{1}{s^2-2\alpha s+\Omega^2}\left[s^2+\left(K(s)-2\alpha 1s+\Omega^2\right)\right]$$

and the characteristic equation is

$$s^2+(K(s)-2\alpha)s+\Omega^2=0 \tag{36}$$

If K(s) is constant and $K>2\alpha$, then the two roots are stable, a familiar example of stabilization by derivative control.

But the real question is: how does the stability vary and how does the effectiveness of feedback control change as the time delay increases? If τ is not too large, and K(s) is taken to be constant, control to stabilize the system is still possible but only in a restricted range of K, a result most clearly displayed by the root locus plot, Figure 10. For this plot, τ=0.1, referred to the normalized

period $T=2\pi$. When $\tau\pm0$.... When $\tau\pm0$, the plot has many branches, most of which eventually end up in the right half plane so the closed-loop system becomes unstable.

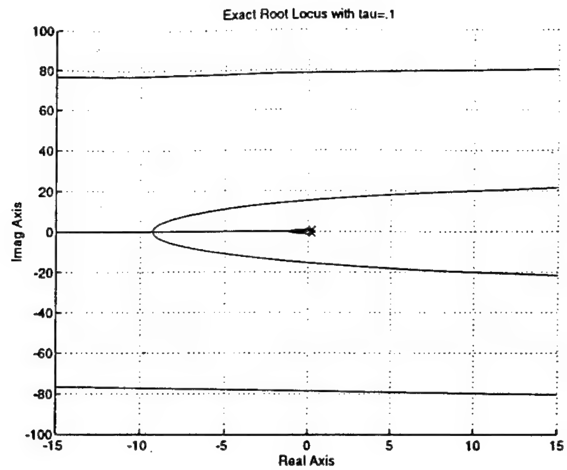


Figure 10

As the time delay increases with an unstable oscillator, feedback stabilization becomes more difficult and virtually impossible when τ is of the order of the period of the oscillator. We have carried out simulations using various sorts of controllers, including state space control, observer control, lead compensation and a Smith regulator. None work. Hence there is a basic problem of controlling an unstable system having a time delay.

That conclusion is not new, and probably not surprising. The puzzling feature in the present context is that some experimental success has been achieved using straightforward strategies, to control instabilities in combustion chambers having time delays associated with internal processes. Two questions arise:

1) In view of the reasoning summarized above, why has success been possible?

2) Will this success translate to larger systems?

It seems that one must admit that the case of a single linear oscillator does not represent well the situation in a combustor. There are at least two deficiencies already mentioned:

1) the motions in a combustor are not linear; and

2) it is misleading to consider only a single mode.

Therefore we are led once again to conclude that to understand fully the potential for application of active feedback control to combustion systems, we must investigate nonlinear behavior. In this program, analysis of nonlinear multi-modal (i.e. multi-dimensional) systems possessing time lags is continuing.

2.5 Active Control of a Combustor Using Pulsed Injection of a Secondary Fuel Supply

Central to practical applications of active feedback control of a combustion chamber are issues of sensing and actuation. In this work we have used only pressure as the sensed variable, measured at only one location. Early in the program we decided to investigate the possibility of using pulsed injection of secondary fuel as the means of actuation. There are two reasons for this choice:

1) At the present time, using a secondary fuel supply as the means of control seems by far the most likely basis for actuation in full-scale systems because that seems to be the most efficient and direct way of affecting the combustion processes responsible for the instabilities. It is highly unlikely that generating pressure waves by speakers, pistons, etc. could be successfully used in operational combustors.

2) Steady or modulated secondary fuel supply has been used previously in laboratory tests with some success, but the amount of fuel used has been usually a substantial fraction of the total fuel flow. Pulsing offers the prospect of reducing the flow used for control.

All tests have been conducted in the Caltech dump combustor constructed and operated for many years under funding from AFOSR. The experiments were conducted largely by a visitor from Germany who completed his undergraduate thesis with this work (Knoop, 1996). The only modifications have been provision for fuel injection at the dump plane, into the recirculation zone, or into the shear layer upstream of the step (Figure 11); and the addition of the apparatus required to supply the fuel. Figure 12 is a diagram of the equipment and instrumentation.

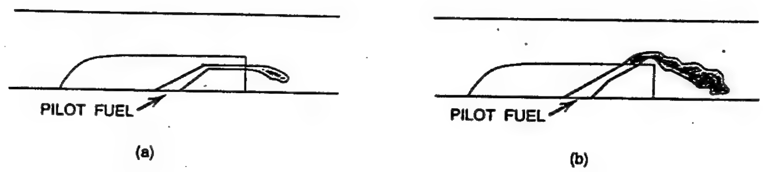


Figure 11

One advantage of using this combustor is that it is well-characterized. Figure 13 is an example of the stability regions in the coordinates average speed V_d past the step and equivalence ratio ϕ (Sterling 1987). All of the initial tests were conducted for a steady operating condition within the unstable region. Results have been reported by Knoop (1996). Some success was achieved in reducing the amplitudes of oscillations, but the significant accomplishment followed upon Knoop's re-discovery of hysteresis on the lean side of the instability region

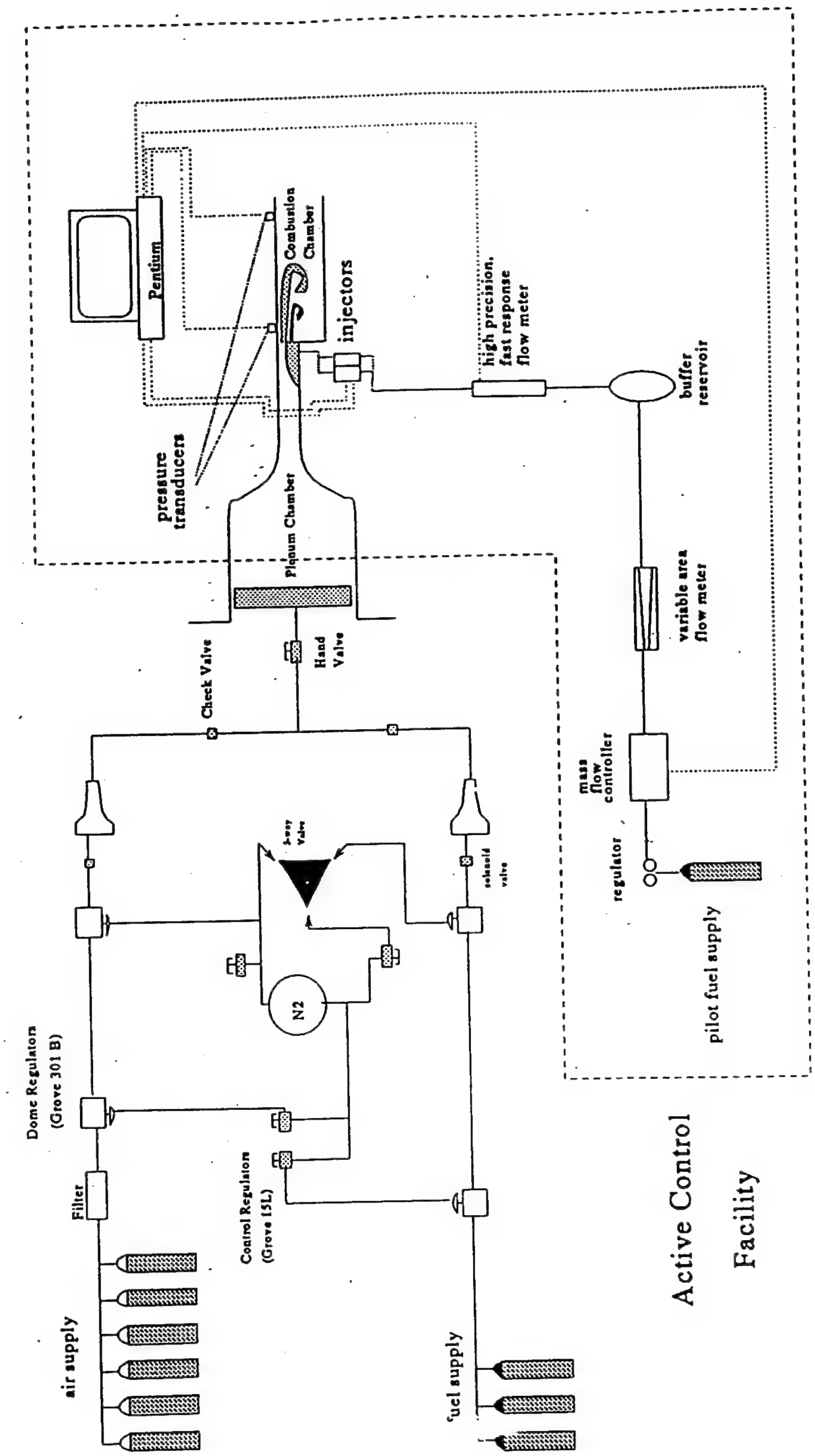

Pentium
Combustion Chamber
injectors
high precision, fast response flow meter
buffer reservoir
pressure transducers
Plenum Chamber
variable area flow meter
Check Valve
Hand Valve
mass flow controller
regulator
pilot fuel supply
N2
Dome Regulators (Grove 301 B)
Filter
Control Regulators (Grove 15L)
air supply
fuel supply
Active Control Facility

Figure 12

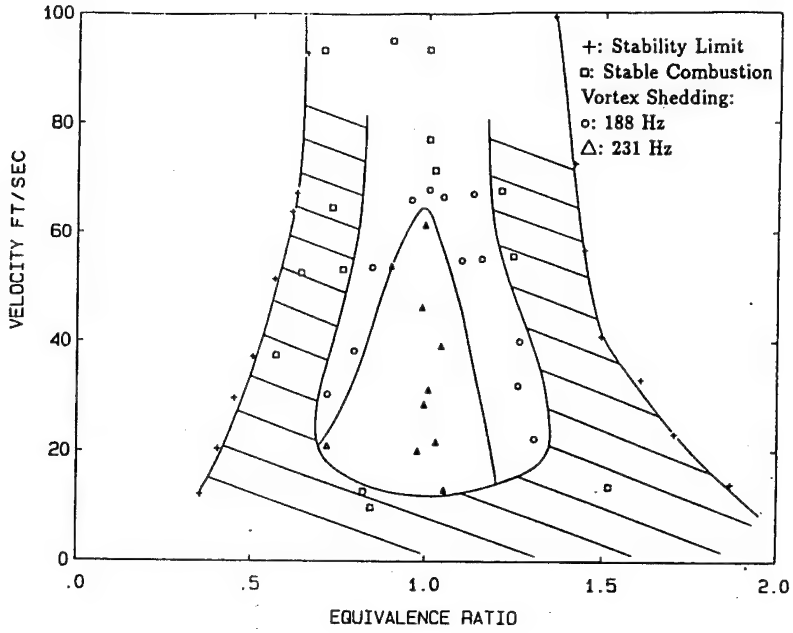


Figure 13

The existence of hysteresis in this combustor was first found by Smith (1985) and investigated further by Sterling (1987). Because the emphasis in those works was on other characteristics of the combustor, the details and implications of hysteresis were not investigated. The significant point here is that a bifurcation diagram having a subcritical bifurcation point and a turning point (cf. Figure 6) implies hysteresis. A stylized sketch more closely illustrating the behavior in the dump combustor is shown in Figure 14. The lower ranch suggests small non-zero pressure oscillations and the upper branch represents stable limit cycles. For the dump combustor, V_d is essentially constant for this diagram so the hysteresis loop degenerates to a line in the $V_d - \phi$ plane of Figure 14.

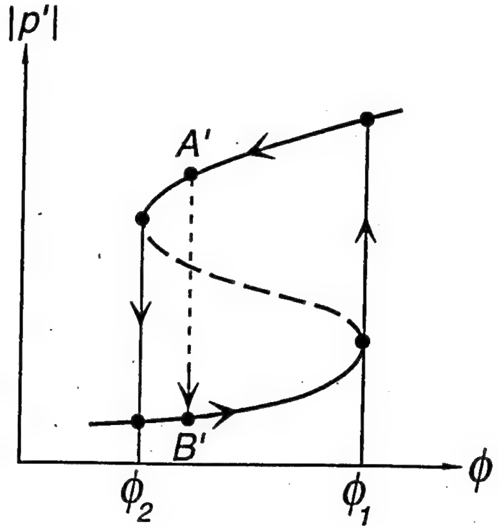


Figure 14

'Hysteresis' means here that the current dynamical state of the system depends on its history. If the equivalence ratio lies between ϕ_2 and ϕ_1, the amplitude of the pressure oscillation depends on the direction from which ϕ was approached, as indicated by the arrows. As ϕ is increased from an initial value less than ϕ_2, the amplitude is low until $\phi = \phi_1$, when a jump occurs and the system executes a relatively high-amplitude limit cycle. On the other hand if ϕ is decreased from a value within the region of instability, the amplitude of the limit cycle remains high until $\phi = \phi_2$ and the transition is made to a low amplitude motion.

The idea of taking advantage of hysteresis to exercise active control is not new; recently it has been applied to problems of surge and stall in compressors (Fu, J.-H. 1988; Abed et al, 1993). Suppose that the combustor is in a limit cycle, say at the point A' in Figure 13. Then if a short disturbance or pulse is applied, $|p'|$ will either increase or decrease momentarily. If the bifurcation diagram remains applicable during this transient period, an increase of $|p'|$ should decay to the upper branch. A decrease of $|p'|$ remaining above the dashed unstable branch should also return to the stable branch. However, a sufficiently large pulse may produce the transition to the point B' on the lower branch.

When that strategy was tried, the transition to the lower branch was soon demonstrated. The process has been confirmed in many subsequent tests. Figure 15 is a plot of a measured hysteresis loop and the transition $A \rightarrow B$ actually achieved. A time-trace of the event is shown in Figure 16
. The dashed line indicates the pulse of injected fuel. In this case two pulses were injected, partly an artifact of the injection system.

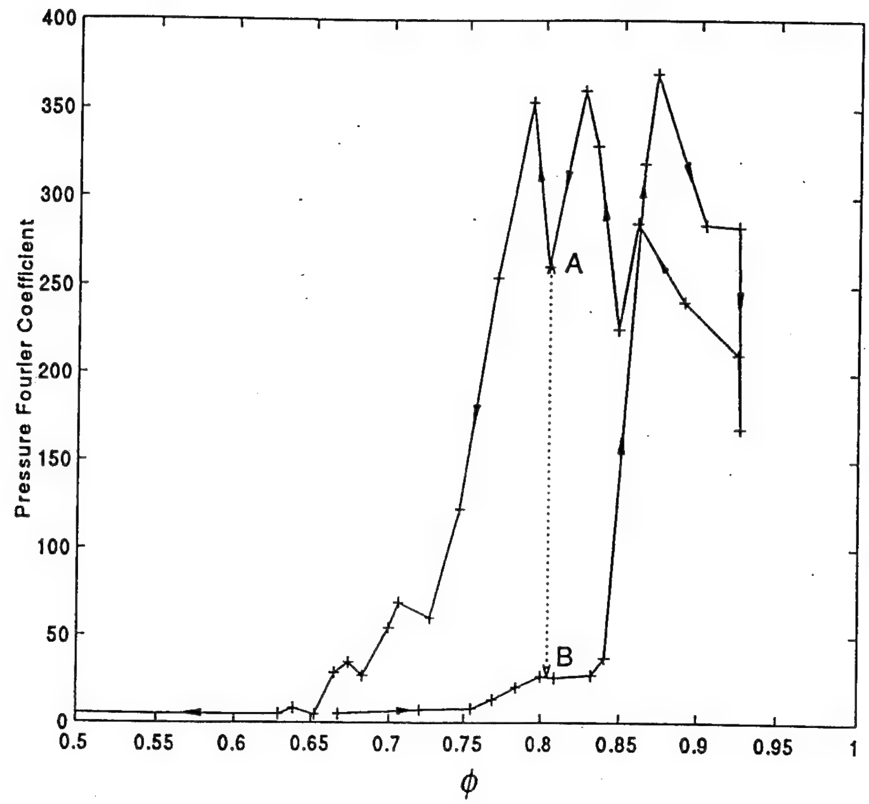


Figure 15

Figure 17 shows the region of hysteresis established experimentally a few months ago. These results show that for this combustor, the region of stable operation can be extended from the left hand part of the diagram to the right hand edge of the shaded region. This is a substantial increase in the range of values of equivalence ratio.

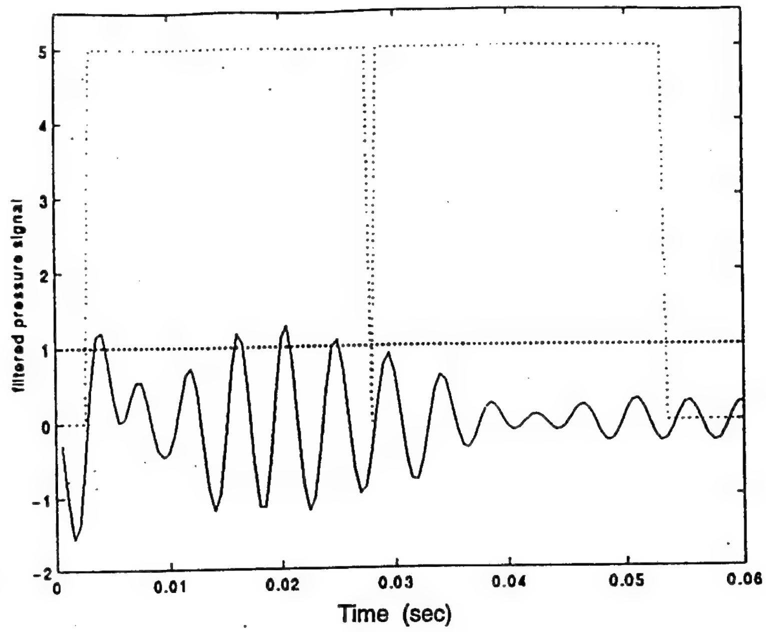


Figure 16

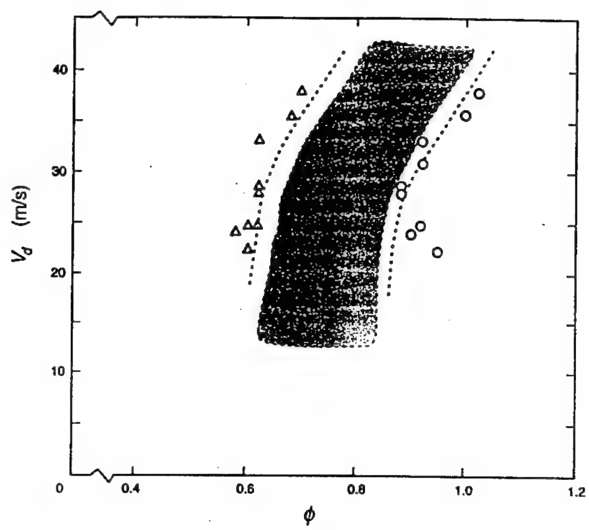


Figure 17

References

Abed, E. H., Houpt, P. K., Hosny, W. M. (1993) "Bifurcation Analysis of Surge and Rotating Stall in Axial Flow Compressors," ASME *Journal of Turbomachinery*, Vol. 115, pp. 814-824.

Baum, J. D. and Levine, J. N. (1982) "Numerical Techniques for Solving Nonlinear Instability Problems in Solid Rocket Motors," *AIAA Journal,* Vol. 20, No. 7, pp. 955-961.

Burnley, V. S. (1996) "Nonlinear Combustion Instabilities and Stochastic Sources," Ph.D. Thesis, Caltech, Div. of E. & A.S.

Burnley, V. S. and Culick, F. E. C. (1996) "Some Dynamics of Acoustic Oscillations With Nonlinear Combustion and Noise," *Proceedings, Fourth International Conference on Special Topics in Combustion,* Stockholm, May 1996.

Chu, B.-T. and Kovásznay, L. S. G. (1958) "Nonlinear Interactions in a Viscous Heat-Conducting Compressible Gas," *Journal of Fluid Mechanics,* Vol. 3, No. 5, pp. 494-514.

Clavin, P., Kim, J. S., and Williams, F. A. (1994) "Turbulence-Induced Noise Effects on High-Frequency Combustion Instabilities," Combustion Science and Technology, Vol. 96, pp. 61-84.

Culick, F. E. C., Paparizos, L., Sterling, J. D. and Burnley, V. (1992) "Combustion Noise and Combustion Instabilities in Propulsion Systems," in *Proceedings of the AGARD Conference on Combat Aircraft Noise,* AGARD CP 512.

Culick, F. E. C. (1994) "Some Recent Results for Nonlinear Acoustics in Combustion Chambers," AIAA Journal, Vol. 32, No. 1, pp. 146-169.

Culick, F. E. C. (1996) "A Note on Ordering Perturbations and the Insignificance of Linear Coupling in Combustion Instabilities," submitted to *Combustion Science and Technology.*

Fu, J.-H. (1988) "Bifurcation Control and Feedback Stabilization, Ph.D. Dissertation, University of Maryland, Department of Electrical Engineering.

Green, W. D. (1990) "Triggering of Longitudinal Combustion Instabilities in Rocket Motors," M. S. Thesis, The Pennsylvania State University.

Jahnke, C. C. and Culick, F. E. C. (1994) "An Application of Dynamical Theory to Nonlinear Combustion Instabilities," *Journal of Propulsion and Power*, Vol. 10, No. 4, pp. 508-517.

Kendrick, D. W. (1995) "An Experimental and Numerical Investigation Into Reacting Vortex Structures Associated With Unstable Combustion" Ph.D. Thesis, Caltech, Div. of E. & A.S.

Knoop, P. (1996) "A Comparison of H_2, N_2 and CH_4 as Pilot Fuel for Suppression of Self-Sustained Pressure Oscillations in a Dump Combustor" Final Thesis (Diplomarbeit) Technische Universität, München and California Institute of Technology.

Knoop, P., Culick, F. E. C. and Zukoski, E. E. (1996) "Extension of the Stability of Motions in a Combustion Chamber by Nonlinear Active Control Based on Hysteresis," submitted to *Combustion Science and Technology*.

Lighthill, M. J. (1953) "On Sound Generated Aerodynamically, Part I: General Theory" *Proc. Roy. Soc.*, A211, pp. 564-587.

Smith, D. A. (1985) "An Experimental Study of Acoustically Excited, Vortex Driven, Combustion Instability Within a Rearward Facing Step Combustor," Ph.D. Thesis, Caltech, Division of E. & A. S.

Sterling, J. D. (1987) "Longitudinal Mode Combustion Instabilities in Air Breathing Engines," Ph.D. Thesis, Caltech, Div. of E & A.S.

Zsak, T. W. (1993) "An Investigation of the Reacting Vortex Structures Associated With Pulse Combustion," Ph.D. Thesis, Caltech, Div. of E. & A.S.

3. Publications and Conference Presentations

Knoop, P. (1996) "A Comparison of H_2, N_2 and CH_4 as Pilot Fuel for Suppression of Self-Sustained Pressure Oscillations in a Dump Combustor" Final Thesis (Diplomarbeit) Technische Universität, München and California Institute of Technology.

Knoop, P., Culick, F. E. C. and Zukoski, E. E. (1996) "Extension of the Stability of Motions in a Combustion Chamber by Nonlinear Active Control Based on Hysteresis," submitted to *Combustion Science and Technology*.

Burnley, V. S. (1996) "Nonlinear Combustion Instabilities and Stochastic Sources," Ph.D. Thesis, Caltech, Div. of E. & A.S.

Burnley, V. S. and Culick, F. E. C. (1996) "Some Dynamics of Acoustic Oscillations With Nonlinear Combustion and Noise," *Proceedings, Fourth International Conference on Special Topics in Combustion,* Stockholm, May 1996.

Isella, G. Seywert, C., Culick, F. E. C. (and Zukoski, E. E. (1996) "A Further Note in Active Control of Combustion Instabilities, Based on Hysteresis." Submitted to *Combustion Science and Technology.*

Culick, F. E. C. (1996) "A Note on Ordering Perturbations and the Insignificance of Linear Coupling in Combustion Instabilities," submitted to *Combustion Science and Technology.*

Culick, F. E. C. (1996) "On the Roles of Approximate Methods of Analysis and Numerical Simulations for Investigation of Combustion Instabilities in Solid Propellant Rockets," St. Petersburg.

Culick, F. E. C. (1996) "A Note on Ordering Perturbations and the Insignificance of Linear Coupling in Combustion Instabilities," Stockholm.

4. Personnel

Faculty

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E. E. Zukoski

Graduate Students

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G. Isella
W. Lin
C. Seywert

Undergraduate Students

R. Jarosiewicz
P. Knoop (visitor from Germany)

5. Collaborations With Industry and Other Organizations

Partly in connection with other research programs (see the last paragraph of Section 2) we have continuing contacts with groups at other universities and industrial firms for reasons directly related to the program reported here:

Universities:

Ecolé Centrale de Paris (Professor S. Candel)
Georgia Tech (Professor B. T. Zinn)
Pennsylvania State University (Professor V. Yang)
Université Aix-Marseille (Professor P. Clavin)
University of California, San Diego (Professor F. A. Williams)
University of Padova (L. Cossalter)

University of Rome (C. Bruno)
University of Washington (Professor D. Pratt and P. Malte)
Vanderbilt University (Professor A. Mellor)

Industrial Firms

Allison/Rolls Royce (Dr. D. Smith)
ENEL Combustion Laboratory, Pisa, Italy (Dr. G. Benelli)
Solar Turbines (Dr. Ken Smith)
United Technologies Research Center (Drs. T. Rosfjord; J. McVey)